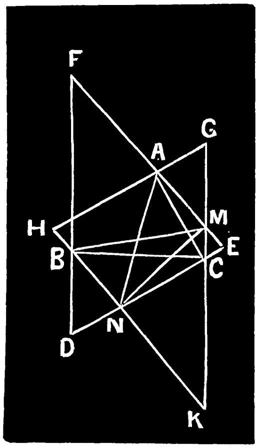
II. Solution by PROPOSER.

Let ABC be the given triangle area $\triangle$; DEF, GHK the circumscribed triangles; FE, GK intersecting in M; DC, HK intersecting in N. Join BM, AN, MN. The triangles DEF, GHK are equiangular. $\angle BAM = \angle BCM = \angle ACN = \angle ABN =$ a right angle.



$\therefore$ A circle passes through A, M, C, B and A, C, N, B, respectively.

$\therefore \angle AMB = \angle ACB = \angle ANB$.

$\therefore AM = BN$ and is parallel to it, $MN = AB$ and is parallel to it.

$\therefore AF = NK$, $ME = HB$, $\therefore FE = HK$ and $DEF = GHK$. Let A=area of DEF, then $2A = EC.AC + FA.AB + DB.BC + \triangle = b^2\cot A + c^2\cot B + a^2\cot C + \triangle = c^2\cot A + a^2\cot B + b^2\cot C + \triangle$.

$\therefore 4A = [a^4 + b^4 + c^4 - bc(b^2+c^2) - ac(c^2+a^2) - ab(a^2+b^2) + 4\triangle^2]/2\triangle$.

Also solved by F. D. Posey, San Mateo, California; L. E. Newcomb, Los Gatos, California; and J. Scheffer, Hagerstown, Md.

MECHANICS.

165. Proposed by O. W. ANTHONY, DeWitt Clinton High School, New York City.

Find the approximate form of a tower of circular cross section 1000 feet high and having a radius of lower base 20 feet, and so constructed that all the parts of the structure shall be subject to the same stress, due to the weight of the part of the tower above.

III. Solution by CHRISTIAN HORNUNG, A. M., Professor of Mathematics, Heidelberg University, Tiffin, O.

Let w=the weight per unit of volume; s=the stress (pressure per square inch); y=the radius of cross section at distance x from top.

Then πy^2=the area of cross section at distance x from top, and, by the conditions of the problem, the increment of the area for any increment of x multiplied by s, must equal the increment of the weight, we get, in the limit, $\frac{dy}{dx} = \frac{w}{2s}y$. Integrating, we find $y = ce^{(w/2s)x}$. Now since $y=20$ when $y=1000$, $c = 20e^{-(1000w/2s)}$.

$\therefore y = 20e^{(w/2s)(x-1000)}$, which, by assigning proper values to w and s, will give the form of an element of the lateral surface of the tower. If we call $w=\frac{1}{10}$ and $s=20$, the radii of the tower at different elevations would be as follows: At 100 feet, 12.13 feet; at 200, 7.36; at 300, 4.46; at 400, 2.70; at 500, 1.64; at 600, .996; at 700, .604; ot 900, .222; at 990, .1417; and at top, .1348 feet or 1.6 inch.

This result will fulfill the required condition as to stress everywhere except near the top.

166. Proposed by G. B. M. ZERR, A. M., Ph D., Parsons, W. Va.

If a gravitating particle of mass m be placed at the point (a, b, c) prove

that the work required to move a particle of unit mass from the point (x, y, z) to an infinite distance is $m[(x-a)^2+(y-\beta)^2+(z-c)^2]^{-\frac{1}{2}}$. Prove also that

$$\left(\frac{d^2}{dx^2}+\frac{d^2}{dy^2}+\frac{d^2}{dz^2}\right)\{m[(x-a)^2+(y-b)^2+(z-c)^2]^{-\frac{1}{2}}\}=0,$$

except when $x=a$, $y=b$, $z=c$.

Solution by S. A. COREY, Hiteman, Iowa.

Let $r=[(x-a)^2+(y-b)^2+(z-c)^2]^{-\frac{1}{2}}$=distance from point (a, b, c) to point (x, y, z). The gravitating force exerted between the two particles will then be m/r^2. As the gravitating force is always directed along r, the work required to move the unit mass from the point (x, y, z) to an adjacent point $(x+dx, y+dy, z+dz)$ by any path would be $(m/^2)dr$. The total work done as r is increased from r to infinity would be $m\int_r^\infty \frac{dr}{r^2}$, or m/r, as requirod.

$$\text{As } m\frac{d^2}{dx^2}.\frac{1}{r}=m\,\frac{3\,(x-a)^2-r^2}{r^5},\quad m\frac{d^2}{dy^2}.\frac{1}{r}=m\,\frac{3\,(y-b)^2-r^2}{r^5},$$

$$m\frac{d^2}{dz^2}.\frac{1}{r}=m\,\frac{3\,(z-c)^2-r^2}{r^5},\quad m\left(\frac{d^2}{dx^2}+\frac{d^2}{dy^2}+\frac{d^2}{dz^2}\right).\frac{1}{r}=0,$$

for all values of r except $r=0$, *i. e.*, except when $x=y$. $y=b$, $z=c$.

Also solved by G. W. Greenwood, B.A. (Oxon); M. E. Graber, A.B.; W. W. Landis; and Proposer.

GROUP THEORY.*

1. [Miscellaneous No. 144, in January, 1904, p. 21.] Proposed by L. E. DICKSON, Ph. D., The University of Chicago.

Burnside, in his systematic search for all simple groups of orders from 661 to 1092 (*Proc. Lond. Math. Soc.*, 1895, pp. 333-338) overlooked the orders $2^3.3^2.11$ and $1008=2^4.3^2.7$. The former is immediately excluded. Discuss the latter.

Solution by W. BURNSIDE, The Croft, Bromley Road, Catford, England.

With regard to the second [order 1008], it is clear that the supposition that the group is simple, with 8 subgroups of order 7, leads to the contradiction of a group of degree 8 with operations of order 21. In the only other case, viz., 36 subgroups of order 7, each self-conjugate in a subgroup of order 28, the group must be expressible as a *primitive* group of degree 36, in which the subgroup of order 28, which leaves one symbol unchanged, has a self-conjugate operation of order 2, which leads to an obvious contradiction.

*See page 100.